1. Očekivanje za binomnu raspodjelu s parametrima n i p:

$$\begin{aligned}\mathbb{E}(X) &= \sum_{k=0}^{n} k\binom{n}{k}p^k q^{n-k} = \sum_{k=1}^{n} k\frac{n\cdots(n-k+1)}{k!}p^k q^{n-k} \\ &= np\sum_{k=1}^{n}\frac{(n-1)\cdots(n-k+1)}{(k-1)!}p^{k-1}q^{n-k} \\ &= np\sum_{l=0}^{n-1}\frac{(n-1)\cdots(n-l)}{l!}p^l q^{(n-1)-l} = np\sum_{l=0}^{n-1}\binom{n-1}{l}p^l q^{(n-1)-l} \\ &= np(p+q)^{n-1} = np.\end{aligned}$$

2. Očekivanje za Puasonovu raspodjelu s parametrom λ:

$$\mathbb{E}(X) = \sum_{n=0}^{\infty} n\frac{\lambda^n}{n!}e^{-\lambda} = \lambda e^{-\lambda}\sum_{n=1}^{\infty}\frac{\lambda^{n-1}}{(n-1)!} = \lambda e^{-\lambda}e^{\lambda} = \lambda.$$

3. Očekivanje za geometrijsku raspodjelu s parametrom p:

$$\mathbb{E}(X) = \sum_{n=1}^{\infty} nq^{n-1}p = p\frac{d}{dq}\sum_{n=0}^{\infty}q^n = p\frac{d}{dq}(1-q)^{-1} = p(1-q)^{-2} = \frac{1}{p}.$$

4. Direktan račun za nalaženje disperzije za binomnu raspodjelu s parametrima n i p:

$$\begin{aligned}\mathbb{E}[X(X-1)] &= \sum_{k=0}^{n} k(k-1)\binom{n}{k}p^k q^{n-k} \\ &= \sum_{k=2}^{n} k(k-1)\frac{n(n-1)(n-2)\cdots 2\cdot 1}{k!(n-k)!}p^k q^{n-k} \\ &= n(n-1)p^2\sum_{k=2}^{n}\frac{(n-2)\cdots 2\cdot 1}{(k-2)!(n-k)!}p^{k-2}q^{n-k} \\ &= n(n-1)p^2\sum_{j=0}^{n-2}\frac{(n-2)!}{j!(n-2-j)!}p^j q^{n-2-j} = n(n-1)p^2.\end{aligned}$$

$$\mathbb{E}[X^2] = \mathbb{E}[X(X-1)+X] = \mathbb{E}[X(X-1)] + \mathbb{E}[X] = n(n-1)p^2 + np$$

$$\mathbb{D}(X) = \mathbb{E}[X^2] - (\mathbb{E}X)^2 = n(n-1)p^2 + np - (np)^2 = np - np^2 = npq.$$

5. Disperzija za Puasonovu raspodjelu s parametrom λ:

$$\begin{aligned}\mathbb{E}[X(X-1)] &= \sum_{n=0}^{\infty} n(n-1)\frac{\lambda^n}{n!}e^{-\lambda} = \lambda^2 e^{-\lambda}\sum_{n=2}^{\infty}\frac{\lambda^{n-2}}{(n-2)!} \\ &= \lambda^2 e^{-\lambda}\sum_{k=0}^{\infty}\frac{\lambda^k}{k!} = \lambda^2 .\end{aligned}$$

$$\mathbb{D}(X) = \mathbb{E}[X^2]-(\mathbb{E}X)^2 = \mathbb{E}[(X(X-1)]+\mathbb{E}[X]-(\mathbb{E}X)^2 = \lambda^2+\lambda-\lambda^2 = \lambda .$$

6. Disperzija za geometrijsku raspodjelu s parametrom p:

$$\begin{aligned}\mathbb{E}(X^2) &= \sum_{n=1}^{\infty} n^2\mathbb{P}(T=n) = \sum_{n=1}^{\infty} n^2 q^{n-1}p = \sum_{n=1}^{\infty} n(n-1)q^{n-1}p + \sum_{n=1}^{\infty} nq^{n-1}p \\ &= pq\sum_{n=2}^{\infty} n(n-1)q^{n-2} + \mathbb{E}(T) = pq\frac{\partial^2}{\partial q^2}\sum_{n=2}^{\infty} q^{n-2} + \mathbb{E}(T) \\ &= pq\frac{2}{(1-q)^3} + \mathbb{E}(T) = \frac{2q}{p^2} + \frac{1}{p}.\end{aligned}$$

$$\mathbb{D}(X) = \frac{2q}{p^2} + \frac{1}{p} - \frac{1}{p^2} = \frac{1-p}{p^2}.$$

7. Očekivanje i disperzija za uniformnu raspodjelu na intervalu (a, b):

$$\mathbb{E}X = \int_{-\infty}^{\infty} xf(x)\,dx = \int_a^b \frac{x}{b-a}\,dx = \frac{1}{b-a}\frac{x^2}{2}\Big|_a^b = \frac{a+b}{2}$$

$$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_a^b \frac{x^2}{b-a}\,dx = \frac{1}{b-a}\frac{x^3}{3}\Big|_a^b = \frac{a^2+ab+b^2}{3}$$

$$\mathbb{D}(X) = \mathbb{E}[X^2]-(\mathbb{E}X)^2 = \frac{a^2+ab+b^2}{3} - \left(\frac{a+b}{2}\right)^2 = \frac{(a-b)^2}{12} .$$

8. Očekivanje i disperzija za eksponencijalnu raspodjelu λ:

$$\mathbb{E}X = \int_{-\infty}^{\infty} xf(x)\,dx = \int_0^{\infty} \lambda e^{-\lambda}\,dx = -xe^{-\lambda x}\big|_0^{\infty} + \int_0^{\infty} e^{-\lambda x}\,dx = 0 + \frac{e^{-\lambda x}}{-\lambda}\Big|_0^{\infty}$$
$$= \frac{1}{\lambda}$$
$$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^{\infty} x^2\lambda e^{-\lambda x}\,dx = \ldots = \frac{2}{\lambda^2}$$

$$\mathbb{D}(X) = \quad \mathbb{E}[X^2] - (\mathbb{E}X)^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

9. Očekivanje i disperzija za normalnu $\mathcal{N}(0,1)$ raspodjelu:

$$\mathbb{E}X = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx = -\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\Big|_{-\infty}^{\infty} = 0$$

$$\mathbb{D}(X) = \quad \mathbb{E}[X^2] - (\mathbb{E}X)^2 = \mathbb{E}[X^2] - 0 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2}}\,dx =$$
$$\frac{1}{\sqrt{2\pi}} \left[-xe^{-\frac{x^2}{2}}\Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx \right] = \frac{1}{\sqrt{2\pi}}[0 + \sqrt{2\pi}] = 1$$

Ako $X : \mathcal{N}(\mu, \sigma^2)$ onda $\quad Z = \dfrac{X - \mu}{\sigma} : \mathcal{N}(0,1)$, pa slijedi

$$0 = \mathbb{E}Z = \mathbb{E}\left[\frac{X-\mu}{\sigma}\right] = \frac{1}{\sigma}\mathbb{E}X - \frac{\mu}{\sigma} \qquad \Longrightarrow \mathbb{E}X = \mu$$

$$1 = \mathbb{D}\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^2}\mathbb{D}(X) \implies \mathbb{D}(X) = \sigma^2.$$

10. Košijeva raspodjela nema očekivanje jer integral $\displaystyle\int_{-\infty}^{+\infty} |x| \frac{1}{\pi(1+x^2)} dx$ ne konvergira.

Dovoljno je uočiti

$$\int_0^{\infty} \frac{x\,dx}{\pi(1+x^2)} = \frac{1}{2\pi} \log(1+x^2)\Big|_0^{\infty} = \infty \qquad .$$